

Vacuum stability, neutrinos, and dark matter

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Abstract

Motivated by the discovery hint of the Standard Model (SM) Higgs mass around 125 GeV at the LHC, we study the vacuum stability and perturbativity bounds on Higgs scalar of the SM extensions including neutrinos and dark matter (DM). Guided by the SM gauge symmetry and the minimal changes in the SM Higgs potential we consider two extensions of neutrino sector (Type-I and Type-III seesaw mechanisms) and DM sector (a real scalar singlet (darkon) and minimal dark matter (MDM)) respectively. The darkon contributes positively to the β function of the Higgs quartic coupling λ and can stabilize the SM vacuum up to high scale. Similar to the top quark in the SM we find the cause of instability is sensitive to the size of new Yukawa couplings between heavy neutrinos and Higgs boson, namely, the scale of seesaw mechanism. MDM and Type-III seesaw fermion triplet, two nontrivial representations of $SU(2)_L$ group, will bring the additional positive contributions to the gauge coupling g_2 renormalization group (RG) evolution and would also help to stabilize the electroweak vacuum up to high scale.

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I. INTRODUCTION

The mechanism of electroweak symmetry breaking has been searched for decades in past accelerators and on-going experiments at the Large Hadron Collider (LHC). In the standard model(SM), a scalar $SU(2)_L$ doublet is responsible for the symmetry breaking. The intrinsic problems associated the SM has intrigued the expectation of new physics at TeV scale. However, no evidence of new physics has been found so far at the LHC with $\sqrt{s} = 7$ TeV. At the same time, both the ATLAS and CMS collaborations[1, 2] have reported on the search of SM Higgs with $\sim 5 \text{ fb}^{-1}$ data and the results show a first hint of SM Higgs with $m_h = 125 \pm 1 \text{ GeV}$. This motivates us to study its implications in vacuum stability and perturbativity bounds within the extension of the SM including neutrino physics and dark matter (DM).

In the SM, the stability of the vacuum is tightly related with the mass of physical Higgs since the quartic self-coupling λ is connected with m_h , $m_h = \sqrt{2\lambda}v$ ($v = 246 \text{ GeV}$). If m_h is too small, the radiative corrections, mainly from the top quark's contribution, can drive λ negative, induce a false and deep minimum at large field values and destabilize the electroweak vacuum. In terms of no clue to new physics beyond the SM at the LHC, the analysis of the stability for $m_h = 125 \text{ GeV}$ can give useful hints on the structure of ultraviolet scale where the new physics may come in.

On the other hand, non-collider experimental results confront the SM with two major puzzles: neutrino masses and dark matter. The phenomenon of neutrino oscillations show that at least two neutrinos have nonzero but small masses located around sub-eV scale. Evidences from astrophysics and cosmology have pointed out that the ordinary baryonic matter is not the dominant form of material in the Universe. Rather, about 23% of energy density of the Universe is non-luminous and non-absorbing matter, called dark matter (DM).

Enclosing dark matter and the massiveness of neutrino into SM may have effects on the Higgs sector for the analysis of stability of vacuum. Although the exact nature of dark matter and neutrino mass is still unknown and their interactions with SM particle vary for different models, there exist several guidelines for our purpose to analyze the vacuum stability. Based on SM gauge structure and changing the SM Higgs potential in a controllable way, the model should introduce new particles as less as possible to make the analysis of stability possible and necessary.

This paper is organized as follows. In section II we introduce two scenarios of the neutrino and the DM respectively, and explain why the our studied frameworks change minimally the scalar potential of the SM. Then we analyze the vacuum stability and perturbativity bounds in these frameworks. In section IV we give a summary and conclude our results.

II. THE FRAMEWORKS OF NEUTRINO AND DARK MATTER

A. The Type-I and Type-III seesaw mechanisms

Within the context of the SM, no Dirac mass term of neutrino can be written due to the absence of right-handed neutrino fields. If the neutrinos are Majorana fermions, that is, the lepton number is no longer a conserved quantity, one can write a dimension-five operator which is relevant to the neutrino masses [3]

$$\mathcal{L}_{eff} = \frac{y_{ab}}{M}(l_{L\alpha}H)(l_{L\beta}H) + \text{h.c.}, \quad (1)$$

where $l_{L(\alpha,\beta)}$ are the $SU(2)_L$ leptonic doublets with the flavor indices $\alpha, \beta = e, \mu, \tau$, and H is the Higgs field. This operator violates lepton number by two units ($\Delta L = 2$), and hence M corresponds to the lepton number breaking scale. After electroweak symmetry breaking, the Higgs field develops vacuum expectation value (VEV), $\langle H \rangle = v/\sqrt{2}$ with $v = 246$ GeV, and the neutrino masses $m_\nu \sim yv^2/M$ are generated. In order to obtain the neutrino masses at sub-eV scale one either assume $M \gg v$ with a sizable couplings y_{ab} or assume M is achievable in the collider experiments with highly suppressed couplings y_{ab} .

One of the most popular approaches to generate the neutrino masses corresponding to Eq. (1) at tree level is the so-called "seesaw mechanism". In this kind of models the neutrino masses are suppressed by a large factor due to the masses of heavy sectors. The heavy fields are either fermions or scalars, therefore, three variations of seesaw mechanisms are introduced. In the so-called Type-I [4] and type-III [5] seesaw mechanism models, the leptonic $SU(2)_L$ singlet and triplet fermions are introduced respectively. Instead of introducing new fermions to the SM, the Type-II seesaw model [6] uses a $SU(2)_L$ triplet scalar carrying a hypercharge $Y = 2$ to give neutrino masses through the new Yukawa interactions between the new triplet scalar and the SM leptonic doublet fields. This triplet scalar brings additional six parameters to the scalar potential which makes the analysis of vacuum stability unclear

and complicated. So for the purpose of this paper, we concentrate on Type-I and Type-III seesaw mechanisms and discuss their effects on the vacuum stability bounds.

Type-I seesaw mechanism: One adds N $SU(2)_L$ singlet fermions to the SM particle content, usually these particles are treated as the right-handed neutrinos ν_{R_i} ($i = 1, \dots, N$). Since ν_R fields do not carry any SM quantum numbers, one can write the relevant lagrangian in the neutrino sector as

$$\mathcal{L}_\nu = Y_{\nu_{\alpha i}} \bar{l}_\alpha \tilde{H} \nu_{R_i} + \frac{1}{2} M_{R_i} \overline{(\nu_R)^c}_i \nu_{R_i} + \text{h.c.}, \quad (2)$$

where $\tilde{H} = i\sigma_2 H^*$, $Y_{\nu_{\alpha i}}$ is the Dirac Yukawa couplings, c is the charge conjugation, and M_{R_i} are the Majorana masses in the diagonal basis of right-handed neutrinos. After block diagonalizing the mass matrix of left and right handed neutrinos, one obtained the standard seesaw formula for the effective light neutrino mass matrix,

$$m_{\nu_{\alpha\beta}} = - \sum_{i=1}^N \frac{M_{D_{\alpha i}} M_{D_{i\beta}}^T}{M_{R_i}}, \quad (3)$$

here M_D is the Dirac mass matrix of neutrinos. Radiative corrections from Y_ν can contribute the Higgs effective potential and have an effect on the vacuum stability.

Type-III seesaw mechanism: Instead of the exchange of heavy fermion singlet fields in the Type-I seesaw model, Type-III seesaw mechanism is mediated by heavy fermions Σ_R which have zero hypercharge and are transformed as a triplet in the adjoint representation under the $SU(2)_L$ gauge group. The triplet can be written in the tensor form,

$$\Sigma_R = \begin{pmatrix} \Sigma_R^0/\sqrt{2} & \Sigma_R^+ \\ \Sigma_R^- & -\Sigma_R^0/\sqrt{2} \end{pmatrix} \equiv \frac{\Sigma_R^i \sigma^i}{\sqrt{2}}, \quad (4)$$

where σ^i are the Pauli matrices and $\Sigma_R^\pm = (\Sigma_R^1 \mp i\Sigma_R^2)/\sqrt{2}$. One can also write the charge conjugated form of Σ_R ,

$$\Sigma_R^c = \begin{pmatrix} \Sigma_R^{0c}/\sqrt{2} & \Sigma_R^{-c} \\ \Sigma_R^{+c} & -\Sigma_R^{0c}/\sqrt{2} \end{pmatrix}. \quad (5)$$

Note that Σ_R^c are left-handed fields. With the definition $\Sigma \equiv \Sigma_R + \Sigma_R^c$, one find the diagonal neutral fields of Σ are Majorana fermions while off-diagonal charged fields are Dirac fermions.

The general Lagrangian involving Σ fields is given by

$$\mathcal{L}_\Sigma = Tr[\bar{\Sigma} i \not{D} \Sigma] - \frac{1}{2} Tr[\bar{\Sigma} M_\Sigma \Sigma] - \overline{l_{L\alpha}} \sqrt{2} Y_{\Sigma_{\alpha i}}^\dagger \Sigma_i \tilde{H} - H^T \epsilon^T \bar{\Sigma}_i \sqrt{2} Y_{\Sigma_{\alpha i}} l_{L\alpha}, \quad (6)$$

where ϵ is the anti-symmetric tensor, Y_Σ is the Dirac-neutrino Yukawa coupling matrix, and M_Σ is the Majorana mass matrix of heavy fermion triplets. After the Higgs develops VEV, the neutral lepton mass matrix in the basis of $(\nu_L, \Sigma^0)^T$ can be written as

$$\begin{pmatrix} 0 & vY_\Sigma^T/2\sqrt{2} \\ vY_\Sigma/2\sqrt{2} & M_\Sigma/2 \end{pmatrix}. \quad (7)$$

In analogous to the Type-I seesaw mechanism, the effective light neutrino mass matrix $m_{\nu_{\alpha\beta}}$ is obtained with the substitutions $Y_\nu \rightarrow Y_\Sigma$ and $M_R \rightarrow M_\Sigma$. Besides the Yukawa interaction, the triplets in type-III seesaw model have gauge interactions and will contribute to the running of gauge coupling, additionally, changing the running of λ indirectly.

B. The darkon and minimal dark matter

There are convincing evidences from cosmology and astrophysics that 23% of the energy density of the Universe is provided by dark matter [7]. The quest for the nature of the missing component boost the investigations on both theoretical and experimental sides. One of the popular DM candidates is the weakly interacting massive particle (WIMP) which is stable on cosmological time scale and a thermal relic of the Big Bang. Typically, one of the simplest way to justify the stability of DM is to impose a Z_2 parity symmetry, for example the R-parity in supersymmetry. In order to study the vacuum stability bounds of Higgs sector in the extension of SM including neutrino and DM sectors, we choose two models of DM which minimally change the SM Higgs potential and keep the direct relation between Higgs quartic-coupling λ and the electroweak vacuum v .

The darkon: The SM with an additional real singlet scalar S [8], called darkon. To stabilize the darkon being a good WIMP candidate, one introduce a discrete Z_2 symmetry into the model. S is odd under the Z_2 transformation while all the SM particles are even. The Lagrangian involving S reads

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{m_0^2}{2}S^2 - \frac{\lambda_S}{4}S^4 - \lambda_{SH}S^2 H^\dagger H. \quad (8)$$

After electroweak symmetry breaking, the Lagrangian \mathcal{L}_S becomes

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{m_0^2 + \lambda_{SH}v^2}{2}S^2 - \frac{\lambda_{SH}}{2}S^2 h^2 - \lambda_{SH}vS^2 h, \quad (9)$$

where we identify h the physical Higgs boson. This is one kind of Higgs-portal models, the interactions between the darkon and Higgs boson play an important role in determining

the relic abundance of DM [9, 10] (also see Fig. 1) and the invisible decay width of Higgs boson [11, 12]. Recent study [13] shows that darkon with $M_S > 80$ GeV or at the resonant region around 62 GeV is still allowed by both invisible decay and XENON100 constraints [14].

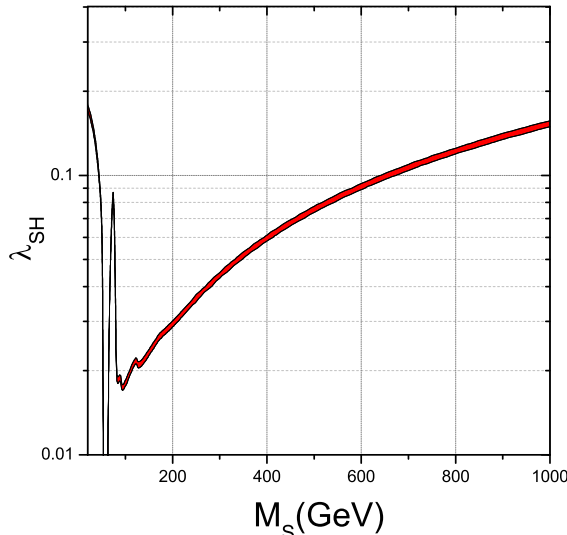


FIG. 1. Constraint on M_S and λ_{SH} from relic density, $\Omega_{\text{cdm}} h^2 = 0.111 \pm 0.006$ [7]. Recent study [13] shows that darkon with $M_S > 80$ GeV or at the resonant region around 62 GeV is still allowed by both invisible decay and XENON100 constraints.

The minimal dark matter: The authors of [15] first proposed the idea of "minimal dark matter". They take the assumption of the existence of only SM gauge symmetry as a guidance and do not introduce extra features (some discrete symmetry, for example) to guarantee the stabilization of DM candidate. In [15] they found an extra electroweak multiplet field χ which carries minimal SM quantum numbers and has one lightest neutral component after quantum corrections, and furthermore, no operator can be written for the χ decay such that neutral component of χ is a viable DM candidate and stable particle. MDM also satisfy all the direct/indirect dark matter search experiments. It was found that the minimal construction of such field (MDM) χ is a "fermionic $SU(2)_L$ quintuplet with hypercharge $Y = 0$ ". χ is a vector field with respect to $SU(2)_L$ symmetry and hence the theory is anomaly free. Since χ transforms as **5** representation under $SU(2)_L$, its neutral component only interacts with the SM particles via Weak and gravity forces. Therefore, it is a WIMP candidate, and has

no free parameter. The mass of MDM is 9.6 TeV which is determined by the relic density of DM [16].

It would be interesting to study the impact on the vacuum stability bounds of the Higgs boson within the above frameworks of the SM extensions including neutrinos and DM. We study the vacuum stability and perturbativity bounds in the combined four scenarios of SM extension: (1) Type-I seesaw + darkon, (2) Type-I seesaw + MDM, (3) Type-III seesaw + darkon, and (4) Type-III seesaw + MDM in the following section.

III. VACUUM STABILITY AND PERTURBATIVITY

We begin with the brief introduction of vacuum stability and perturbativity in the SM. It is known (see [17–26] and refs therein) that the SM Higgs potential is modified by high order quantum corrections. For Higgs quartic coupling λ , its β function, β_λ , receives positive contributions from scalars and gauge bosons and negative contributions from fermions. Especially, when λ is too large as an input at the electroweak scale, the self-interaction will drive $\lambda(\mu)$ to a non-perturbative region and may confront the theory with Landau pole or triviality. And when λ is too small, it will become negative due to the large top quark Yukawa coupling at certain energy scale Λ , the scale corresponds to the instability of the electroweak vacuum and the new physics will appear at this scale. With fixed m_h , the scale Λ is sensitive to the top quark mass and strong coupling constant (see Fig. 2).

Before the LHC’s running, with m_h as a free and unknown parameter, some relevant works on vacuum stability has been studied in extensions of SM, including the scalar singlet [27–29], Type-I seesaw [30, 34], Type-II seesaw mechanism [35], and Type-III seesaw mechanism [36]. And recent related works with m_h around 125 GeV are [30] with Type-I seesaw, [31, 32] in SM, and [33] with scalar dark matter. To our knowledge, no combination or comparison of both seesaw mechanism and DM extension of SM has been considered. With the suggestive hint at the LHC, in what follows, we take the SM Higgs mass $m_h = 125$ GeV as an input and analyze the stability in different frameworks.

Before studying of stability in the combination scenarios of neutrino and DM two sectors, we shall firstly compare the qualitative difference in each sector and show the quantitative effects of individual parameters. Then we show how the $\lambda(\mu)$ is changed with both neutrino and DM extensions.

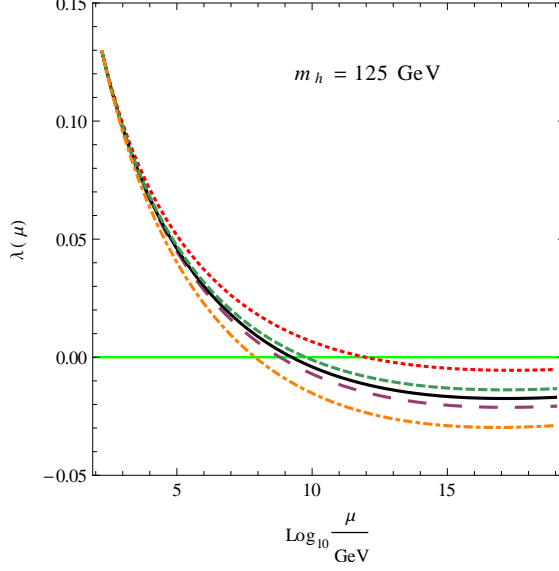


FIG. 2. Running of $\lambda(\mu)$ in standard model. Black solid line is plotted with $m_t = 173.2$ GeV and $\alpha_s(M_Z) = 0.1184$. Dotted and Dot-Dashed lines shows the effects of changing mass of m_t , for $m_t = 171.4$ GeV and $m_t = 175.0$, respectively. Two lines, closer to the solid one, display the effects of different value for $\alpha_s(M_Z)$, dashed one for $\alpha_s(M_Z) = 0.1198$ and long dashed for $\alpha_s(M_Z) = 0.1170$.

A. SM + Type-I/Type-III seesaw mechanism

Both the Type-I and Type-III seesaw mechanisms have the new Yukawa couplings (Y_ν and Y_Σ respectively) with Higgs bosons. These bring the additional corrections to the β function of λ :

$$\Delta\beta_{\lambda_I} = \frac{1}{(4\pi)^2} \left[-4 \text{Tr} Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + 4\lambda \text{Tr} Y_\nu Y_\nu^\dagger \right] \quad (10)$$

and

$$\Delta\beta_{\lambda_{III}} = \frac{1}{(4\pi)^2} \left[-20 \text{Tr} Y_\Sigma Y_\Sigma^\dagger Y_\Sigma Y_\Sigma^\dagger + 12\lambda \text{Tr} Y_\Sigma Y_\Sigma^\dagger \right], \quad (11)$$

for Type-I seesaw and Type-III seesaw respectively. For simplicity, we study the case that there is a mass hierarchy of light neutrino masses ($m_{\nu_3} > m_{\nu_2} > m_{\nu_1}$, for example) due to the hierarchy of Yukawa couplings, and hence, the masses of the heavy neutrino sectors are degenerate. In this case, we consider only one sizable Yukawa corresponding to one heavy neutrino sector which will affect the RG evolution of λ^1 . The triplet fermions in Type-III

¹ We assume the light neutrino masses are around $\sqrt{\Delta m_{\text{atm}}^2} \approx 0.05$ eV in our analysis.

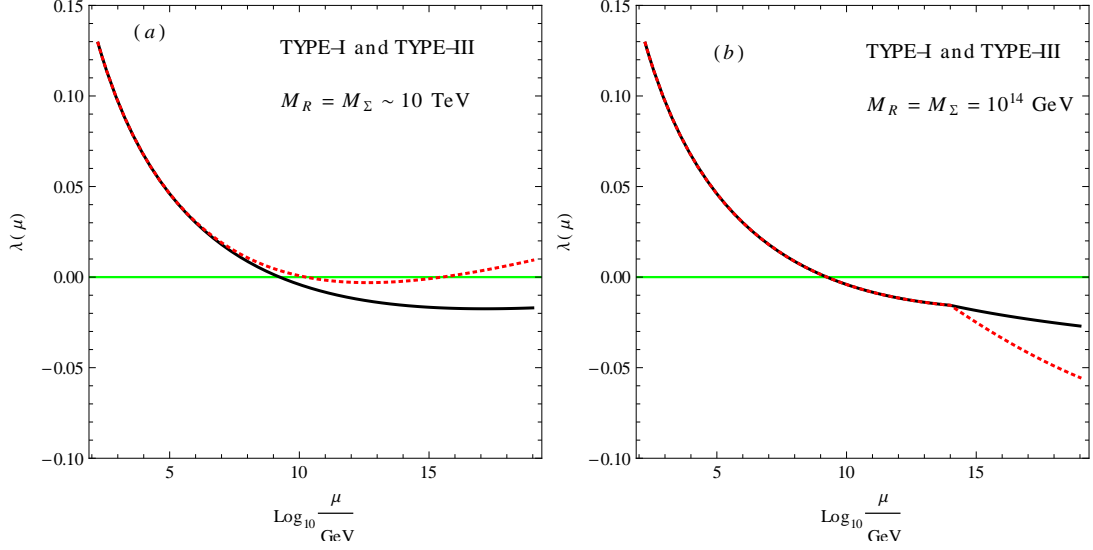


FIG. 3. Comparison between type-I and type-III seesaw mechanisms for different mass scales of heavy neutrino. (a) $M_R = M_\sigma = 10 \text{ TeV}$, the black solid line shows the running of $\lambda(\mu)$ in type-I seesaw and the red dotted one for type-III seesaw. (b) $M_R = M_\sigma = 10^{14} \text{ GeV}$, the black solid line show the running of $\lambda(\mu)$ in type-I seesaw and the red dotted one for type-III seesaw. The threshold effect can be easily seen at $\mu = 10^{14} \text{ GeV}$.

seesaw will also change the $SU(2)_L$ gauge coupling g_2 RG equation with the modification,

$$\Delta\beta_{g_{2\text{III}}} = \frac{1}{(4\pi)^2} \frac{4n}{3}, \quad (12)$$

here n is the generations of heavy triplets, and we assume $n = 3$. The runnings of these Yukawa coupling are also taken into account, governed by their β functions in the Appendix.

In Fig. 3 we compare the RG evolutions in the cases of Type-I and Type-III seesaw mechanisms extension with different mass scales of heavy fermions. As shown in the figures, when the mass scale of the heavy neutrinos is smaller than 10^{14} GeV , these neutrion Yukawa couplings is small and can be ignored. The deviation between Type-I and Type-III seesaw mechanisms is caused dominantly by the gauge coupling running when $M_R = M_\Sigma = 10 \text{ TeV}$ in (a) of Fig. 3, and by fermion degrees of freedom and when $M_R = M_\Sigma = 10^{14} \text{ GeV}$ in (b) of Fig. 3.

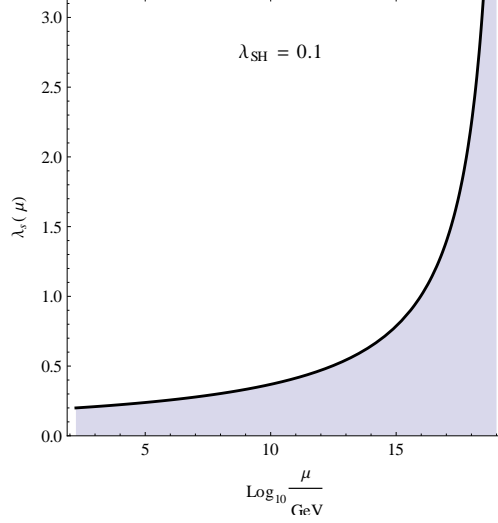


FIG. 4. Perturbativity bound for λ_S . From the constraint of relic density, when $M_S \sim 1$ TeV, $\lambda_{SH} \sim \mathcal{O}(0.1)$ shown in Fig. (1). If we further require that $\lambda_S(10^{19}) < \pi$ for perturbativity, then $\lambda_S(M_S) < 0.2$ at the input scale.

B. SM + darkon/MDM

Now we consider the cases of SM extension of dark matter sector. For darkon model, we first scan the parameter space which is allowed by various constraints. The relic abundance for DM density is shown previously in Fig. 1. Since λ_S always receives positive contributions from Higgs boson, we do not have to worry about the stability of S at large field values. Instead, we plot the perturbative bounds for λ_S in Fig. 4 and find that in the range of $0 < \lambda_S < 0.2$ one can satisfy the perturbative bounds up to M_{Pl} with $\lambda_{SH} = 0.1$ which provides the strongest constraint as shown in Fig. 1.

The scalar singlet will modify the β_λ as

$$\Delta\beta_{\lambda_{\text{darkon}}} = \frac{1}{(4\pi)^2} 2\lambda_{SH}^2, \quad (13)$$

accompanied by β functions for darkon sector in the Appendix. We also plot in Fig. 5((a),(b)) to show the sensitivity of λ RG running to λ_S and λ_{SH} respectively, and Fig. 5(c) implies the stabilization holds as long as $\lambda_{SH} \gtrsim 0.031$ for $\lambda_S = 0.1$.

Rather than contributing β_λ directly, the minimal dark matter will change the RG evolution of $SU(2)_L$ gauge coupling g_2 with

$$\Delta\beta_{g_{2\text{MDM}}} = \frac{1}{(4\pi)^2} \frac{20}{3}, \quad (14)$$

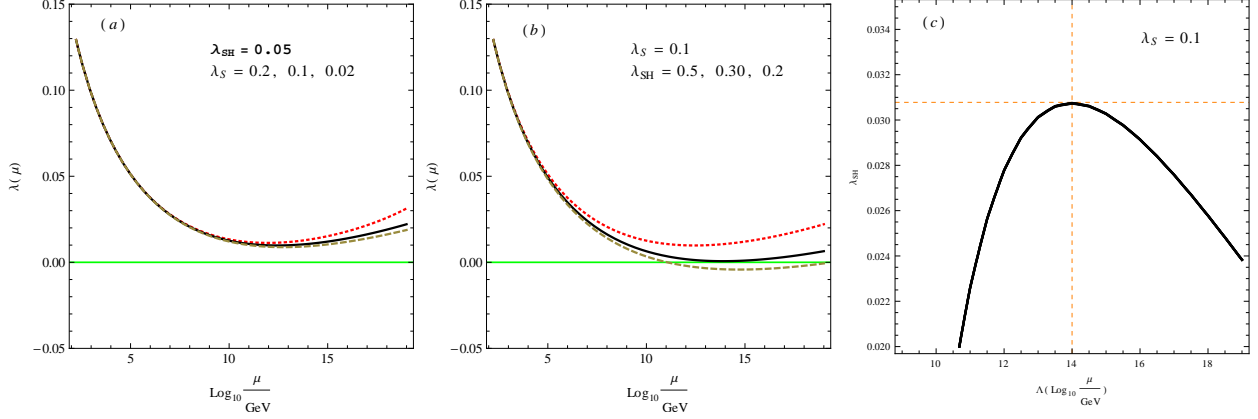


FIG. 5. Effects on running of λ when varying λ_S and λ_{SH} , respectively. (a) Dotted, solid and dashed lines correspond to $\lambda_S = 0.2, 0.1$ and 0.02 , respectively. (b) Dotted, solid and dashed lines correspond to $\lambda_{SH} = 0.5, 0.3$ and 0.2 , respectively. (c) show Λ at which $\lambda(\Lambda) = 0$ for different λ_{SH} , λ_S is fixed here as an example. When $\lambda_S > 0.031$, stability can be preserved up to Planck scale.

and as a result, gives a positive contribution to β_λ indirectly. We note that there is no free parameter in the case of MDM, therefore we plot in Fig. 6 to show the differences between the SM and the extension of darkon and MDM. The positive contribution to $\lambda(\mu)$ is completely due to the growing of gauge coupling $g_2(\mu)$ in MDM scenario. In the case of darkon, the raise of $\lambda(\mu)$ is more for larger λ_{SH} and λ_S while the value of λ_{SH} has to be consistent with the relic abundance of DM.

Confronting the unsolved puzzles of neutrino mass and dark matter in the SM, we study the extensions of SM on the Type-I(III) seesaw and darkon (MDM) with the four combinations in the following.

SM + Type-I/Type-III seesaw + darkon:

The figure is plotted in Fig. 7 which shows that when Yukawa couplings are smaller than $\mathcal{O}(0.1)$, both Type-I and Type-III seesaw are irrelevant due to the forth power suppression of $Y_{\nu,\Sigma}$ in $\beta_{\lambda_{I,III}}$. In other words, the running of $\lambda(\mu)$ is sensitive to the scales of heavy neutrinos as we see in Fig. 7. When the scales of the heavy neutrino sectors are large ($\mathcal{O}(10^{14-15})$ GeV), the Yukawa couplings are sizable and we see the instability of electroweak vacuum occurs around the new neutrino scales. The drop of $\lambda(\mu)$ is sharper for the Type-III seesaw due to more degrees of freedom (triplet fermion) involved in the quartic-coupling box diagram (see Eqs. (10),(11)). The effects due to the growing of gauge coupling g_2 is relatively weaker in these scenarios.

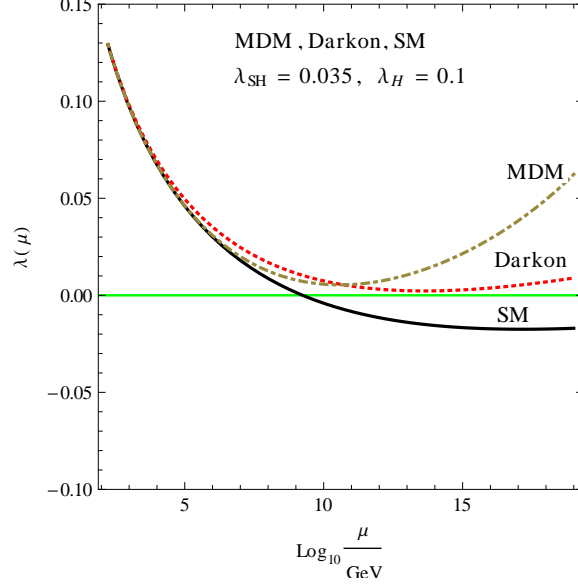


FIG. 6. Comparison between MDM and darkon's effects on the running of $\lambda(\mu)$. Unlike the Darkon case, in MDM extension there is no free parameter after the mass is fixed by relic density and $\lambda(\mu) > 0$ is valid up to Planck scale.

SM + Type-I/Type-III seesaw + MDM:

In these cases the effects of gauge coupling $g_2(\mu)$ running are important to $\lambda(\mu)$. Since the mass of MDM is 9.6 TeV fixed by relic density of DM [16] and the $g_2(\mu)$ starts to receive the additional contribution (Eq. (14)) from MDM at this scale. This will cause the raise of $\lambda(\mu)$ via the growing of $g_2(\mu)$ to compensate for the negative contributions from Yukawa couplings and avoid the instability at high scales. From Fig. 8 we see how the $g_2(\mu)$ modify the evolutions of $\lambda(\mu)$ in Type-I/Type-III plus MDM frameworks. One should notice that one of the conditions of minimal dark matter is the perturbativity of g_2 to be valid up to Planck scale. If the three generation triplet fermions of Type-III seesaw are lighter than 10^8 GeV, the perturbativity of $g_2(\mu)$ will blow up at the scale around $\mathcal{O}(10^{15})$ GeV. The validity of fermionic quintuplet as minimal dark matter may be questioned in this extended scenario. However, the mass spectrum of and the generation numbers of Σ_i will alter the g_2 evolution and relax the perturbativity bound up to M_{Pl} . We will not address this discussion in this paper.

At last, we show how stability can constrain the mass scales in the neutrino sector in Fig. 9. If there is no new physics other than dark matter and neutrino up the Planck scale,

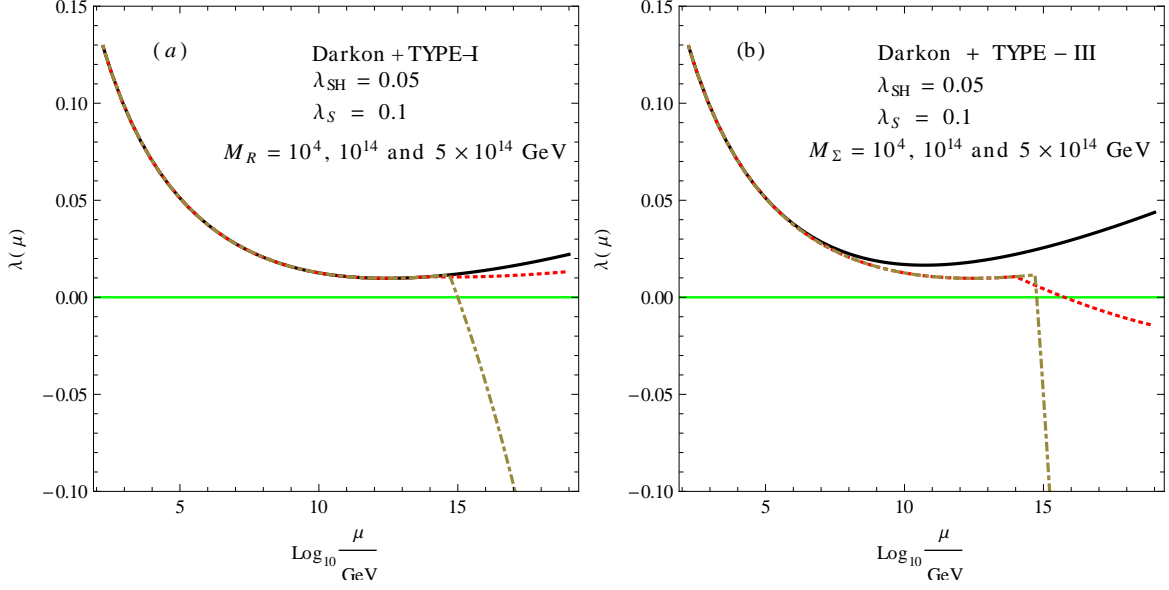


FIG. 7. Combination of Darkon and neutrino in type-I and type-III seesaw mechanisms. The qualitative features of the running of $\lambda(\mu)$ are the same in (a) and (b). And quantitative differences results from the neutrino's contribution to β_λ in two scenarios. In both (a) and (b), solid, dotted and dot-dashed lines represent the effects from different mass scales of heavy neutrinos, 10^4 , 10^{14} and 5×10^{14} GeV, respectively.

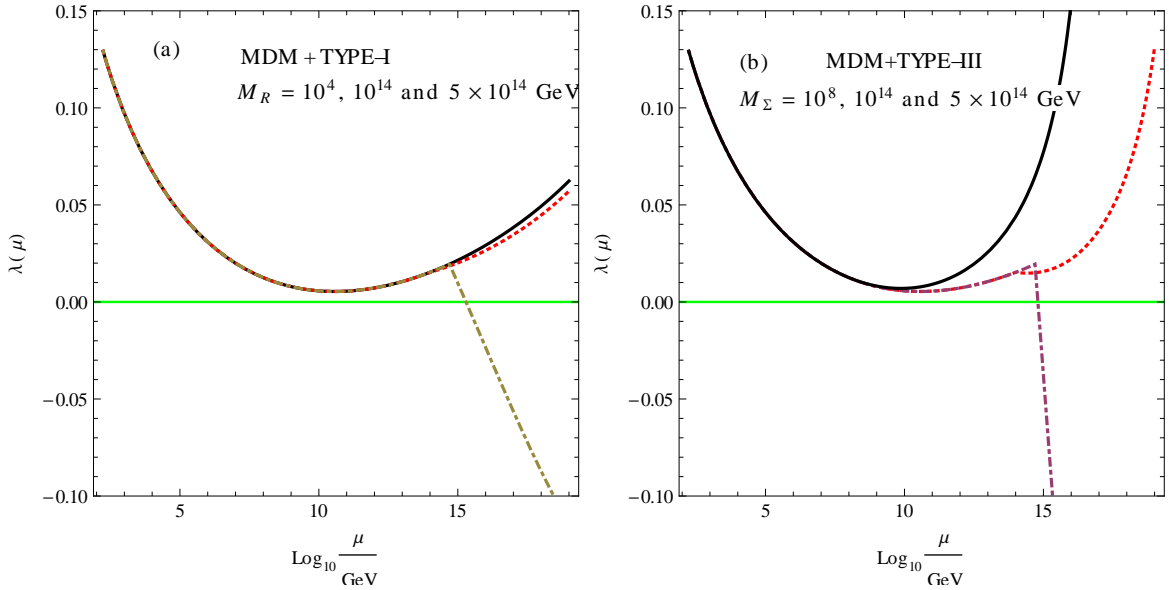


FIG. 8. Combination of MDM and neutrino in type-I and type-III seesaw mechanisms. In both (a) and (b), solid, dotted and dot-dashed lines represent the effects from different mass scales of heavy neutrinos, 10^4 , 10^{14} and 5×10^{14} GeV, respectively.

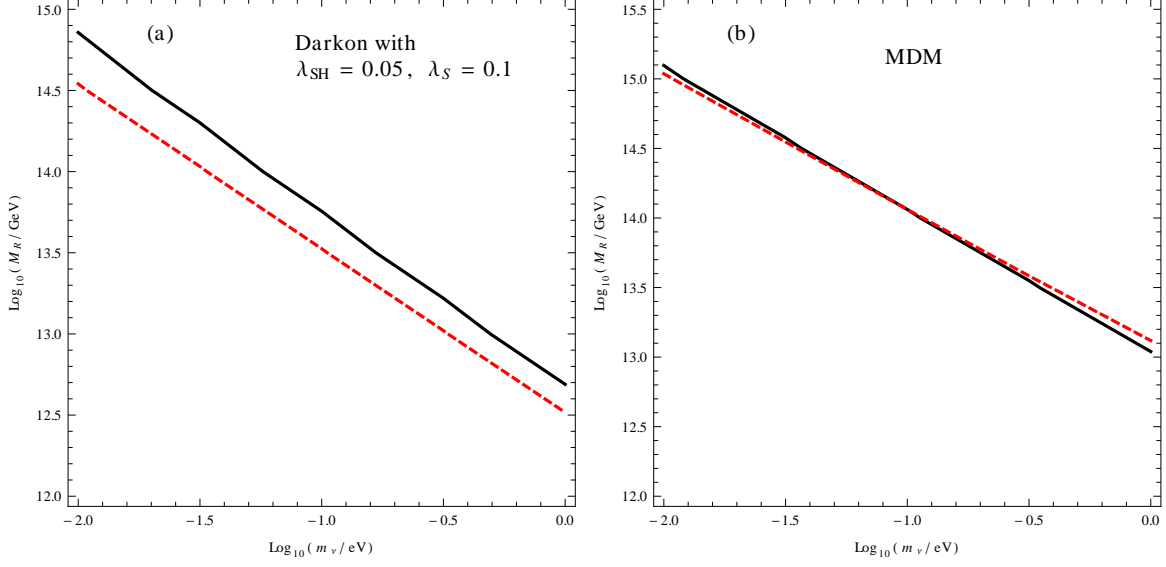


FIG. 9. Constraint on the neutrino sector with the requirement $\lambda(M_{Pl}) > 0$. In both (a) and (b), solid and dashed lines display the contours at $\lambda(M_{Pl}) = 0$ for type-I and type-III seesaw mechanism, respectively. Regions under the line satisfy $\lambda(M_{Pl}) > 0$.

$\lambda(M_{Pl}) > 0$ can put bounds on neutrino part with fixed parameters in DM sector. In (a) of Fig. 9, the contours $\lambda(M_{Pl}) = 0$ are plotted in the framework of Darkon+Type-I/III. Here bounds on Type-III seesaw are more stringent because of the magnitude of its contribution to β_λ as also suggested in Fig. 7. With the combination of MDM+Type-I/III, (b) of Fig. 9 shows the individual bounds. Contour lines are closer than those in (a), a feature of the competition between gauge coupling and Yukawa's contributions.

IV. CONCLUSIONS

We study the stability of the electroweak vacuum within the SM and its extensions of including the Type-I(III) seesaw mechanisms and dark matter candidates (darkon and minimal dark matter) and also discussed the stability bounds on the scales of neutrino sector. Motivated by the possible evidence of the SM Higgs at the LHC, we apply the signal of $m_h = 125$ GeV as the input and investigate the RG evolutions of Higgs quartic-coupling λ in four scenarios: (1) Type-I seesaw + darkon, (2) Type-I seesaw + MDM, (3) Type-III seesaw + darkon, and (4) Type-III seesaw + MDM. The scalar singlet contributes positively to the β function of λ to stabilize the SM vacuum up to high scale. However, similar to

the top quark mass in the SM, the results depend sensitively on the Yukawa couplings $Y_{\nu,\Sigma}$ between the new neutrino sectors and SM Higgs. If $Y_{\nu,\Sigma} \sim \mathcal{O}(1)$, it will bring the instability at the heavy neutrino mass scale. Additionally, minimal dark matter and fermionic triplets of Type-III seesaw mechanism transform nontrivially under the $SU(2)_L$ gauge group, and hence, will boost the growth of gauge coupling $g_2(\mu)$ running. As a result, they will bring the positive contributions to save the stability of Higgs vacuum.

ACKNOWLEDGMENTS

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Appendix A: Renormalization Group(RG) Equations

In this appendix we list the relevant input and RG equations in our numerical evaluation.

$$m_h = 125 \text{ GeV}, \quad m_t = 173.2 \pm 0.9 \text{ GeV}, \quad M_Z = 91.188 \text{ GeV},$$

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007, \quad \alpha(M_Z) = 1/127.926, \quad \sin^2\theta(M_Z) = 0.2312.$$

And λ and y_t are related with m_h and m_t by matching condition [37], respectively.

Standard Model: For the standard model, we collect RG equations at two-loop order for Higgs quartic coupling, top-quark Yukawa coupling and the gauge couplings, λ , y_t and g_i [38–42]. For the Higgs quartic coupling we have

$$\begin{aligned} \beta_\lambda = & \frac{1}{(4\pi)^2} \left[24\lambda^2 - 6y_t^4 + \frac{3}{8} \left(2g^4 + (g^2 + g'^2)^2 \right) + (-9g^2 - 3g'^2 + 12y_t^2) \lambda \right] \\ & + \frac{1}{(4\pi)^4} \left[\frac{1}{48} (915g^6 - 289g^4g'^2 - 559g^2g'^4 - 379g'^6) + 30y_t^6 - y_t^4 \left(\frac{8g'^2}{3} + 32g_s^2 + 3\lambda \right) \right. \\ & + \lambda \left(-\frac{73}{8}g^4 + \frac{39}{4}g^2g'^2 + \frac{629}{24}g'^4 + 108g^2\lambda + 36g'^2\lambda - 312\lambda^2 \right) \\ & \left. + y_t^2 \left(-\frac{9}{4}g^4 + \frac{21}{2}g^2g'^2 - \frac{19}{4}g'^4 + \lambda \left(\frac{45}{2}g^2 + \frac{85}{6}g'^2 + 80g_s^2 - 144\lambda \right) \right) \right]. \end{aligned} \quad (\text{A1})$$

For the top-quark Yukawa coupling we have

$$\begin{aligned} \beta_{y_t} = & \frac{y_t}{(4\pi)^2} \left[\frac{9}{2}y_t^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2 - 8g_s^2 \right] + \frac{y_t}{(4\pi)^4} \left[-\frac{23}{4}g^4 - \frac{3}{4}g^2g'^2 + \frac{1187}{216}g'^4 + 9g^2g_s^2 \right. \\ & \left. + \frac{19}{9}g'^2g_s^2 - 108g_s^4 + \left(\frac{225}{16}g^2 + \frac{131}{16}g'^2 + 36g_s^2 \right) y_t^2 + 6(-2y_t^4 - 2y_t^2\lambda + \lambda^2) \right]. \end{aligned} \quad (\text{A2})$$

And for the gauge couplings $g_i = \{g', g, g_s\}$, we have

$$\beta_{g_i} = \frac{1}{(4\pi)^2} g_i^3 b_i + \frac{1}{(4\pi)^4} g_i^3 \left[\sum_{j=1}^3 c_{ij} g_j^2 - d_i y_t^2 \right], \quad (\text{A3})$$

with

$$b = (41/6, -19/6, -7), \quad c = \begin{pmatrix} 199/18 & 9/2 & 44/3 \\ 3/2 & 35/6 & 12 \\ 11/6 & 9/2 & -26 \end{pmatrix}, \quad d = (17/6, 3/2, 2). \quad (\text{A4})$$

Darkon : In evaluating the effects from darkon's contribution, β functions for darkon [29] itself are also considered. We have at one-loop order

$$\beta_{\lambda_{SH}} = \frac{1}{(4\pi)^2} \left[8\lambda_{SH}^2 + 6\lambda_{SH}\lambda_S - \frac{9}{2}\lambda_{SH}g^2 - \frac{3}{2}\lambda_{SH}g'^2 + 6\lambda_{SH}y_t^2 + 2\lambda_{SH}\lambda \right], \quad (\text{A5})$$

and

$$\beta_{\lambda_S} = \frac{1}{(4\pi)^2} [8\lambda_{SH}^2 + 18\lambda_S^2], \quad (\text{A6})$$

for the running of λ_{SH} and λ_S , respectively.

Neutrino : The running of Yukawa coupling for the neutrino sector is governed by [34, 36, 43]

$$\mu \frac{d}{d\mu} (Y_\nu^\dagger Y_\nu) = \frac{1}{(4\pi)^2} Y_\nu^\dagger Y_\nu \left[6y_t^2 + 2 \text{Tr} (Y_\nu^\dagger Y_\nu) - \left(\frac{9}{10}g_1^2 + \frac{9}{10}g_2^2 \right) + 3Y_\nu^\dagger Y_\nu \right] \quad (\text{A7})$$

for Type-I seesaw and

$$\mu \frac{d}{d\mu} (Y_\nu^\dagger Y_\nu) = \frac{1}{(4\pi)^2} Y_\nu^\dagger Y_\nu \left[6y_t^2 + 3 \text{Tr} (Y_\nu^\dagger Y_\nu) - \left(\frac{9}{10}g_1^2 + \frac{33}{2}g_2^2 \right) + \frac{5}{2}Y_\nu^\dagger Y_\nu \right] \quad (\text{A8})$$

for Type-III seesaw mechanisms at one-loop order. When neutrino Yukawa couplings are not ignorable, they can change the β -function of top-quark Yukawa by

$$\Delta\beta_{y_t} = \frac{1}{(4\pi)^2} [\text{Tr} (Y_\nu^\dagger Y_\nu)] \quad (\text{A9})$$

in Type-I seesaw and

$$\Delta\beta_{y_t} = \frac{1}{(4\pi)^2} [3 \text{Tr} (Y_\nu^\dagger Y_\nu)] \quad (\text{A10})$$

in Type-III seesaw.

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